# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE 2154

## AN ANALYSIS OF THE AUTOROTATIVE PERFORMANCE OF A HELICOPTER POWERED BY ROTOR-TIP JET UNITS

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## SUMMARY

The autorotative performance of an assumed helicopter was studied to determine the effect of inoperative jet units located at the rotor-blade tip on the helicopter rate of descent. For a representative ram-jet design, the effect of the jet drag is to increase the minimum rate of descent of the helicopter from about 1,500 feet per minute to 3,700 feet per minute when the rotor is operating at a tip speed of approximately 600 feet per second. The effect is less if the rotor operates at lower tip speeds, but the rotor kinetic energy and the stall margin available for the landing maneuver are then reduced. Power-off rates of descent of pulse-jet helicopters would be expected to be less than those of ram-jet helicopters because pulse jets of current design appear to have greater ratios of net power-on thrust to power-off drag than currently designed ram jets.

In order to obtain greater accuracy in studies of autorotative performance, calculations involving high power-off rates of descent should include the weight-supporting effect of the fuselage parasite-drag force and the fact that the rotor thrust does not equal the weight of the helicopter.

## INTRODUCTION

The autorotative operation of a helicopter following sudden power failure in flight is recognized as an important design condition. In general, the autorotative rates of descent of conventionally powered helicopters with normal disk loadings $\left(\text{ranging from 2 to } 3\frac{1}{2} \text{ lb/sq ft}\right)$ have proved to be satisfactory to the pilot from the standpoint of safety and controllability. The autorotative rates of descent of helicopters powered with rotor-blade-tip jet units, on the other hand, present a problem to the designers of such aircraft because of the relatively high drag of the jet units when they are inoperative. In this condition, the high "cold" (that is, power-off) drag of the units, acting at high tip velocities, absorbs a relatively large amount of profile-drag power which must be supplied by gravity (or by a high rate of vertical descent for a helicopter of fixed weight).

In order to obtain more quantitative information concerning the effects of the cold drag of the tip jet, the autorotative performance of an assumed helicopter was calculated for several values of jet-unit cold drag coefficients and the results are presented herein.

## SYMBOLS

| | |
|---|---|
| $a$ | slope of curve of section lift coefficient against section angle of attack (radian measure), assumed equal to 5.73 herein |
| $A_j$ | projected frontal area of jet units, square feet |
| $b$ | number of blades per rotor |
| $c$ | blade section chord, feet |
| $c_{d_o}$ | blade section profile-drag coefficient |
| $c_{d_j}$ | drag coefficient of jet units based on frontal area |
| $\Delta c_{d_j}$ | drag coefficient of jet units based on frontal area expressing difference between drag of jet units and that portion of blade between $R_b$ and $R$ |
| $C_T$ | rotor thrust coefficient $\left(\frac{T}{\pi R^2 \rho (\Omega R)^2}\right)$ |
| $\overline{C}_L$ | rotor mean lift coefficient, calculated as $\frac{2}{\frac{1}{3} + \mu^2} \frac{C_T}{\sigma}$ |
| $C_{Q_a}$ | rotor accelerating torque coefficient $\left(\frac{Q_a}{\rho (\Omega R)^2 \pi R^3}\right)$ |
| $C_{Q_d}$ | rotor decelerating torque coefficient $\left(\frac{Q_d}{\rho (\Omega R)^2 \pi R^3}\right)$ |

| | |
|---|---|
| $C_{Q_j}$ | inoperative-jet-drag torque coefficient $\left(\frac{Q_j}{\rho(\Omega R)^2 \pi R^3}\right)$ |
| $D_p$ | parasite drag of helicopter, pounds |
| $(D/T)_o$ | rotor profile drag-thrust ratio |
| $(D/T)_i$ | rotor induced drag-thrust ratio |
| $(D/T)_p$ | parasite drag of helicopter components other than lifting rotors divided by rotor thrust |
| $(D/T)_g$ | drag-thrust ratio of helicopter in autorotative glide |
| $f$ | equivalent-flat-plate area representing helicopter parasite drag, based on unit drag coefficient, square feet $\left(\frac{D_p}{\frac{1}{2}\rho V^2}\right)$ |
| $i$ | angle between rotor thrust vector and a vertical line, degrees |
| $Q_a$ | rotor accelerating torque, pound-feet |
| $Q_d$ | rotor decelerating torque, pound-feet |
| $Q_j$ | torque required to overcome drag of inoperative jet units, pound-feet |
| $R$ | blade radius measured to outboard end of jet unit, feet |
| $R_j$ | blade radius measured to center line of jet unit, feet |
| $R_b$ | blade radius measured to inboard end of jet unit, feet |
| $T$ | rotor thrust, pounds |
| $V$ | true airspeed of helicopter along flight path, feet per second |
| $V_h$ | horizontal component of true airspeed of helicopter, feet per second |

| | |
|---|---|
| $V_v$ | vertical component of true airspeed of helicopter, feet per second |
| $v$ | induced inflow velocity at rotor (always positive), feet per second |
| $W$ | gross weight of helicopter, pounds |
| $\alpha$ | rotor angle of attack; angle between axis of no feathering and plane perpendicular to flight path, positive when axis is pointing rearward, radians |
| $\alpha_r$ | blade-element angle of attack, measured from line of zero lift, radians $(\theta + \phi)$ |
| $\alpha_{(u_T=0.4)(270^\circ)}$ | blade-element angle of attack at radius at which tangential velocity equals 0.4 tip speed and at $270^\circ$ azimuth position, degrees |
| $\gamma$ | glide-path angle, degrees |
| $\delta$ | mean section profile-drag coefficient for portion of blade between $R_b$ and $R$ |
| $\theta$ | blade-section pitch angle; angle between line of zero lift of blade section and plane perpendicular to axis of no feathering, radians |
| $\lambda$ | inflow ratio $\left(\frac{V \sin \alpha - v}{\Omega R}\right)$ |
| $\mu$ | tip-speed ratio $\left(\frac{V \cos \alpha}{\Omega R}\right)$ |
| $\rho$ | mass density of air, slugs per cubic foot |
| $\sigma$ | rotor solidity $(bc/\pi R)$ |
| $\phi$ | inflow angle at blade element in plane perpendicular to blade-span axis, radians |
| $\Omega$ | rotor angular velocity, radians per second |

## ASSUMED HELICOPTER STUDIED

The helicopter studied was a small, one-place, single-rotor craft. The rotor had two untwisted blades and was powered by a ram jet located at the tip of each blade. Its characteristics are as follows:

| | |
|---|---|
| Weight, $W$, pounds | 600 |
| Blade radius measured to outboard end of jet unit, $R$, feet | 9 |
| Blade radius measured to center line of jet unit, $R_j$, feet | 8.69 |
| Blade radius measured to inboard end of jet unit, $R_b$, feet | 8.38 |
| Rotor solidity, $\sigma$ | 0.05 |
| $f/\pi R^2$ | 0.05 |
| Mass density of air, $\rho$, slugs per cubic foot | 0.002378 |
| Jet-unit outside diameter, inches | 7.5 |
| Disk loading (when hovering), pounds per square foot | 2.36 |

The variation of profile-drag coefficient with angle of attack of the blade of the assumed helicopter is

$$c_{d_o} = 0.0087 - 0.0216\alpha_r + 0.400\alpha_r^2$$

and is representative of well-built blades with smooth and essentially nondeformable surfaces (see reference 1).

The size of the ram-jet unit of the assumed helicopter was decided on the basis that the unit be required to produce sufficient thrust to enable the helicopter to hover at sea-level conditions above ground effect at a cruising tip speed of 600 feet per second and with a small reserve-thrust margin. A value of the ratio of net power-on thrust to cold drag of unity was assumed for the sample jet as being representative of current subsonic ram-jet engines. This ratio, together with an assumed 0.20 cold drag coefficient (based on the maximum frontal area of the jet unit) yielded a net power-on thrust coefficient which, together with the calculated net jet thrust, leads to the size and drag of the units. In view of this procedure, the drag of the jet units of the assumed helicopter is believed to be representative of the minimum value for satisfactory performance with current ram-jet design practice. Current pulse-jet engines, on the other hand, appear to have higher ratios of net power-on thrust to cold drag than ram-jet engines and could therefore be expected to have less power losses in the cold condition than the assumed helicopter.

For the same jet diameter, calculations were made for two different values of the drag coefficient of the inoperative tip units, namely 0.10

and 0.20. The use of the 0.10 drag coefficient was equivalent to assuming a ratio of net thrust to cold drag of two, instead of unity, and would thus represent an improvement of the power-off performance of current jet units. In order to compare the results of these calculations with the power-off performance of conventionally powered rotors, results were also obtained for zero jet drag coefficient.

## METHOD OF ANALYSIS

Although an estimate of the magnitude of the effect of the jet drag on the autorotative performance of the assumed helicopter may be obtained by correcting the helicopter rate of descent for the power loss contributed by the jet units, this method becomes inaccurate when the jet-drag contribution is of the same order of magnitude as the rate of descent of the conventional rotor. In this case, the additional descent speed markedly affects the rotor induced and profile-drag losses, as well as the parasite drag of the fuselage itself. A more refined approach, such as given in references 1 and 2, is required. Additional factors which these references ignore, but which should be considered when high angles of descent are involved, are the vertical component of the fuselage parasite-drag force, which acts to help support the weight of the helicopter, and the fact that the thrust cannot be considered equal to the weight of the helicopter even if the vertical component of the fuselage drag force is ignored. All these factors were taken into account by the analysis used herein to calculate the effect of the jet units on the autorotative performance of the assumed helicopter.

In calculating the solidity, and thus the thrust, of the assumed rotor with tip jet units, the blades were assumed to extend to the outboard end of the jet units. This assumption, which is equivalent to assuming that the lift of the jet units is equal to the lift of the blade area replaced, is adequate for the purposes of the present investigation and would seem to be more correct than ignoring the lift of the jet units altogether.

An outline of the method of analysis as applied to the problem of the autorotation of a jet helicopter is given in the appendix. The vertical power-off descent points were calculated according to the method outlined in reference 3.

Although an experimental check of the rotor theory covering the high values of pitch and inflow combinations involved in the present paper has not been made, the validity of the theory for more moderate values of pitch and inflow combinations has been verified. In particular, a check of the theory has been obtained in autorotation at rates of descent that are reached by present-day conventional helicopters.

The investigations, reported in references 4 and 5, involved a comparison of the autorotative performance of two sets of blades having different amounts of profile drag. The results of the tests indicated that the measured differences in performance were of the same order as that predicted by theory.

## RESULTS AND DISCUSSION

Effect of jet drag coefficient.- The autorotative performance of the assumed helicopter for the various configurations investigated is given in figure 1 in terms of rate of descent plotted against horizontal component of airspeed. Values of thrust, rotor speed, and blade pitch angle corresponding to the various calculated flight speeds are listed in table I.

Figure 1(a) gives the performance of the helicopter operating at a constant value of $\frac{C_T}{\sigma} = 0.055$, which corresponds to operation at a rotor tip speed of 600 feet per second when the rotor thrust is equal to the weight of the helicopter (600 lb for the present case). For the hovering or vertical-flight case, this value of $C_T/\sigma$ is equivalent to a value of rotor mean lift coefficient $\bar{C}_L = 0.33$. $\left(\text{The mean lift coefficient may be computed for any forward speed by means of the relation } \bar{C}_L = \frac{2}{\frac{1}{3} + \mu^2}\frac{C_T}{\sigma}.\right)$

The figure shows that the addition of the cold ram jets to the basic helicopter rotor $\left(\text{that is, } c_{d_j} = 0\right)$ increases the minimum rate of descent from about 1,500 feet per minute to 2,600 feet per minute for $c_{d_j} = 0.10$ or to 3,700 feet per minute for $c_{d_j} = 0.20$. The vertical rate of descent is increased from 2,270 feet per minute to 2,780 feet per minute or 3,740 feet per minute, depending on the drag of the jet units. Thus, the minimum rates of descent in power-off flight of helicopters with tip-located jet units are apt to be very much greater than those with conventionally powered rotors. It should be realized, however, that the detrimental effect of the jet units would be alleviated in part by the increased rotor kinetic energy available to the pilot during landing that is contributed by the mass of the jet units. If, however, by means of sufficiently heavy tip units, enough energy should be supplied to check the extreme rates of descent brought about by the jet drag, there would still be left open the problem of exploration of new landing approach techniques wherein the pilot must approach the ground at unusually high vertical velocities.

For the operating condition of figure 1(a), the autorotating helicopter is noted to have a definite limit to the horizontal speed at which it can travel. This limitation arises from the fact that the horizontal component of the resultant thrust vector, which is available to overcome the horizontal component of the parasite drag, reached a maximum value. A maximum is reached because the magnitude of the thrust vector decreases at a faster rate than is compensated for by the increase in tilt of the vector with increasing flight speeds. The magnitude of the thrust decreases with increasing speed because the vertical component of the parasite drag offsets an increasing part of the helicopter weight; thus a smaller thrust is required to support the helicopter.

Operation at high lift coefficients.- It can be inferred that operation at lower tip speeds will result in a lower rotor drag and in a substantial reduction in the rate of descent of the helicopter over the speed range. Lower tip speeds represent operation at higher mean lift coefficients. The curves of figure 1(b), which represent operation at an approximate mean lift coefficient $\bar{C}_L$ of 0.74 $\left(\frac{C_T}{\sigma} = 0.124\right.$, or a tip speed of 400 fps for 600 lb of rotor thrust$\left.\right)$, show this reduction when compared with the curves for $\bar{C}_L = 0.33$ of figure 1(a). Figure 1(b) shows that, at the higher mean lift coefficient, the minimum rate of descent varies from 1,100 feet per minute for $c_{d_j} = 0$ to 1,450 feet per minute for $c_{d_j} = 0.10$ and to 1,850 feet per minute for $c_{d_j} = 0.20$. Operation at very high mean lift coefficients, which result in more normal rates of descent, is unfortunately not feasible in the usual landing maneuver wherein a flare-out is employed because of the danger of excessive rotor-blade stalling and the resulting loss of rotor speed and thrust. (If no flare-out maneuver is possible with either the total or cyclic-pitch controls, however, as might be the case during a power-off descent under absolutely "blind weather" conditions, it would be best to operate at a mean lift coefficient as close to the stall as possible so that the helicopter would hit the ground at the lowest possible contact velocity.)

The degree to which stall is present during operation at the two values of $\bar{C}_L$ previously discussed may be inferred from figure 2, which gives as a function of $\mu$ the limiting values of $C_T/\sigma$ for inboard stall limits corresponding to airfoil-section stall angles of attack of 12° and 16° as calculated from reference 2. (These stall limits are discussed in detail in reference 1.) The figure shows that the assumed helicopter rotor will be partially stalled over a large part of the speed range in steady autorotative flight if it operates at the larger of the two values of $C_T/\sigma$. (The horizontal velocities corresponding to the $\mu$ values are indicated in the figure for the two $C_T/\sigma$ values under discussion.) Although the limiting amount of inboard stall from the standpoint of the loss of rotor speed and thrust has not been definitely established as yet, the figure indicates that little pull-up margin is available at the higher lift coefficients to decelerate the helicopter during a landing maneuver. Operation at the lift coefficient represented by the curves of figure 1(b) may therefore be considered as an approximate upper limit which cannot be safely exceeded. It follows that the increased rates of descent corresponding to this mean lift coefficient are the minimum that could be expected by the addition of the jet units to the rotor considered in this paper.

Inasmuch as approximate calculations indicate that the rotor thrust might be as much as doubled during a pull-up maneuver from these high rates of descent in order to achieve satisfactory deceleration of the helicopter in a reasonable amount of time, a rotor may be seriously stalled during the pull-up if it is operating close to the stall during steady flight. Thus, the potential benefit of the high rotor kinetic energy available from the tip units would not be sufficient to prevent the helicopter from making a hard landing. Such energy does useful work only when it permits rotor operation at higher than normal values of thrust without a serious loss in rotor speed during the time required to decelerate the helicopter. If appreciable stall is present, the available rotor kinetic energy is dissipated in overcoming the considerably increased blade profile drag. Compared with the case of no stall, therefore, the reduction in rotor speed to the minimum value that could be tolerated would be more rapid; thus the available decelerating time would decrease and the final landing velocity would increase. In addition, stall would prevent the thrust from increasing as rapidly with pitch as it would normally.

An inspection of the blade-pitch values given in table I reveals that if rates of descent corresponding to those shown in figure 1(a) are tolerated, then the collective pitch range of the jet-driven helicopter must be increased over that for conventional helicopters in order to allow operation at high negative pitch angles.

Operation at constant tip speed.- It can also be seen from table I that, as the resultant airspeed is increased, the rotor thrust required to support the fixed weight of the helicopter decreases because of the greater contribution of the vertical component of the parasite drag. Inasmuch as operation at fixed thrust coefficient was assumed, the decrease in thrust at the higher speeds results in a decrease in rotor speed. This decrease in rotor speed results in lower jet and blade profile-drag losses, and, consequently, in lower rates of descent than would be obtained by operating at constant tip speed. This effect can be seen in figure 3 which gives the performance of the helicopter with $c_{d_j} = 0.20$ during operation at a constant tip speed of 400 feet per second, as compared with the curves for $\frac{C_T}{\sigma} = 0.124$ of figure 1(b). The curves of figure 3 show that operation at constant tip speed results in somewhat higher rates of descent than operation at constant mean lift coefficient. It should be realized, however, that the differences shown by the curves would be negligible if, instead of basing both curves on a common tip speed and mean lift coefficient in hovering flight, the constant-tip-speed curve were computed for a tip speed equal to a value reached by operation at constant $C_T/\sigma$ somewhere in the vicinity of 30 miles per hour. (The difference would be more marked in the case of operation at 600 fps as compared with the curves for $\frac{C_T}{\sigma} = 0.055$ of fig. 1(a).) In actual flight, however, the pilot would tend to follow normal piloting procedure of operating at constant, rather than variable, rotor speed. Inasmuch as the rates of descent obtained by the two methods of operation are not significantly different, emphasis is placed in this paper on operation at constant mean lift coefficient because it is easier to calculate and because the operating margin before the occurrence of stall in a pull-up, for example, is made more apparent. Such calculations also show the very significant effects of high-drag jet units on the autorotative rate of descent even when the rotor is operating at optimum conditions of pitch and rotor speed.

Contributions of individual sources of power loss to total rate of descent.- In order to aid in estimating the relative importance of the various drag-producing elements to the total rate of descent, the contributions of each of the individual sources of power loss to the total rate of descent are shown in figures 4(a) and 4(b) for the 0.20 jet-drag-coefficient case and for the rotor mean lift coefficients corresponding to figures 1(a) and 1(b). In general, the induced and blade profile-drag losses are noted to contribute but little to the total rate of descent. For the condition of $\overline{C}_L = 0.33$, the rates of descent at the low airspeeds are high and therefore result in low induced losses so that little additional benefit is realized by travelling at higher airspeeds. Thus, the total-rate-of-descent curve does not show a marked minimum in the 30- or 40-mile-per-hour region as is customary for conventional helicopters. A minimum is present, however, for the condition of $\overline{C}_L = 0.74$, inasmuch as the rates of descent in the low-speed range are lower; thus a significant reduction in induced power is obtained by operation at higher speeds.

The reduction in jet-drag contribution shown in figure 4(a) arises from the fact that, for operation at a constant mean lift coefficient, the rotor speed drops considerably as the forward speed is increased with a consequent reduction in jet profile drag. This effect is smaller for the case of higher $\overline{C}_L$ shown in figure 4(b) (inasmuch as the drop in rotor speed with forward speed is less because of the smaller vertical component of parasite drag) and zero in the case of operation at constant tip speed.

General remarks.- It is apparent from the preceding discussion that the general effect of inoperative tip jet units will be to cause a marked and perhaps dangerous increase in helicopter power-off rates of descent unless proper measures are taken to increase the ratio of net power-on thrust to cold drag by

(1) Increasing the net thrust coefficient. For the ram jet, this increase can be obtained through an increase in the temperature ratio and a reduction in the internal losses of the jet. Such an increase would allow the same thrust to be developed by a unit of smaller diameter. (Higher thrust coefficients are achieved by pulse jets as compared with ram jets, primarily as a result of their ability to produce maximum thrust at relatively lower speeds. This fact, together with roughly equivalent cold drag coefficients, would tend to give the pulse-jet-powered helicopter power-off performance superior to that of the ram-jet helicopter.)

(2) Reducing the cold drag coefficient of the units by refinements in internal and external design. (A reduction in the cold drag coefficient would be obtained primarily by a decrease in drag of the burners. Although a reduction in the external drag of the units, by carefully designing the jet housing or by incorporating the units into the rotor blades, would be just as beneficial, the possibilities for improvement seem much more limited.) The cold drag of the jet units could also be decreased by designing the rotor with a larger solidity than would normally be used for power-on operation. Although such a measure would result in reduced power-on efficiency (for a fixed tip speed), it would allow the rotor to operate at lower tip speeds in the power-off condition and still have an adequate stall margin.

Such measures as jettisoning the tip units as soon as power failure occurs are open to question because of the possibility of dangerous vibration if the units were not released simultaneously, of the danger to people on the ground, and of the danger of excessive control sensitivity resulting from the marked reduction in rotor damping.

Inasmuch as the cold drag of the jet units can only be reduced and not eliminated, the question arises as to what is the maximum autorotative rate of descent that is acceptable to the pilot in steady gliding flight. The solution involves primarily the ease and safety with which the pilot can arrest the helicopter from its high velocity of descent and land it. Thus, the available kinetic energy in the rotor, the margin of mean rotor lift coefficient between the value at which the rotor is effectively stalled and the trim value, and the design of the landing gear all contribute to the pilot's opinion. It would therefore appear logical for the pilot to determine for each design the maximum speed acceptable to him by gradually increasing the rate of descent up to that for the completely power-off condition by means of partial-power descents.

In addition, further studies should be made regarding the compromise between operation at low lift coefficients, which result in high rates of descent but more available rotor kinetic energy, and high lift coefficients, which result in lower rates of descent and lower available rotor kinetic energy. Such studies would involve calculating the autorotative rates of descent corresponding to operation at various mean lift coefficients, the final pull-up velocity corresponding to each of the different descent velocities, and, finally, the optimum pilot pull-up technique which would result in the lowest final pull-up velocity corresponding to a given amount of rotor kinetic energy.

## CONCLUSIONS

On the basis of a study of the autorotative performance of an assumed helicopter powered by rotor-tip jet units, the following conclusions may be made:

1. For a ratio of net power-on thrust to power-off drag of unity (which is representative of current ram jets), the jet unit increases the minimum rate of descent of the helicopter from about 1,500 feet per minute to 3,700 feet per minute when the rotor is operating at a tip speed of approximately 600 feet per second. The effect is less if the rotor operates at lower tip speeds, but the rotor kinetic energy and the stall margin available for the landing maneuver are then reduced.

2. Power-off rates of descent of pulse-jet helicopters would be expected to be less than those of ram-jet helicopters because pulse jets of current design appear to have greater ratios of net power-on thrust to power-off drag than currently designed ram jets.

3. Because the power-off drag of tip jet units could cause a marked and perhaps dangerous increase in the minimum rate of descent of the helicopter, steps should be taken to reduce the power-off drag of the units and to determine the maximum autorotative rate of descent that is acceptable to the pilot in steady gliding flight.

4. In making the analysis, it was found that in order to obtain greater accuracy in studies of autorotative performance, calculations involving high power-off rates of descent should include the weight-supporting effect of the fuselage parasite-drag force and the fact that the rotor thrust does not equal the helicopter weight.

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National Advisory Committee for Aeronautics
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# APPENDIX

## OUTLINE OF METHOD FOR CALCULATING THE AUTOROTATIVE PERFORMANCE OF HELICOPTERS POWERED BY ROTOR-TIP JET UNITS

The calculations required to compute the autorotative glide angle and velocity corresponding to the given design characteristics and to a given tip-speed ratio of a helicopter powered by tip jet units in forward flight are outlined in the following steps (the individual equations were obtained directly or were derived from the equations and methods given in references 1 to 3):

(1) Compute $\lambda$ in terms of $\theta$ from

$$\frac{2C_T}{\sigma a} = \left(t_{3,1}\right)\lambda + \left(t_{3,2}\right)\theta \tag{1}$$

(2) Compute $2C_{Q_a}/\sigma$ and $2C_{Q_d}/\sigma$ as follows:

$$\frac{2C_{Q_a}}{\sigma} = a\left[\left(t_{4,1}\right)\lambda^2 + \left(t_{4,2}\right)\lambda\theta + \left(t_{4,4}\right)\theta^2\right] \tag{2}$$

$$\frac{2C_{Q_d}}{\sigma} = 0.0087\left(t_{5,1}\right) - 0.0216\left[\left(t_{5,2}\right)\lambda + \left(t_{5,3}\right)\theta\right] + 0.400\left[\left(t_{5,5}\right)\lambda^2 + \left(t_{5,6}\right)\lambda\theta + \left(t_{5,8}\right)\theta^2\right] \tag{3}$$

Values for the $t$ constants in equations (1), (2), and (3) are listed in reference 1 for different values of $\mu$.

(3) Inasmuch as the rotor profile-drag losses were calculated on the basis that the blades extend to the tip of the jet units, determine the torque contribution of the jet units with the use of a drag coefficient which represents the difference in drag between the jet units and that part of the blade tip that is replaced by the units. Calculate the jet drag coefficient expressing this drag difference $\Delta c_{d_j}$ from the following relation:

$$\Delta c_{d_j} = c_{d_j} - \delta \frac{c}{A_j} \frac{\left[\frac{1}{2}\mu^2(R - R_b) + \frac{1}{3}\left(R - \frac{R_b{}^3}{R^2}\right)\right]}{\left(\frac{R_j}{R}\right)^2 + \frac{1}{2}\mu^2} \tag{4}$$

(In these calculations $\delta$ was assumed equal to 0.0087.) The torque resulting from the drag of the jet units may then be calculated from the following expression for the torque coefficient:

$$C_{Q_j} = \Delta c_{d_j} \frac{A_j}{\pi R^2} \frac{R_j}{R}\left[\left(\frac{R_j}{R}\right)^2 + \frac{1}{2}\mu^2\right] \tag{5}$$

(4) Substitute the results of step (1) into equations (2) and (3), and substitute the resulting equations into the following relation:

$$\frac{2}{\sigma} C_{Q_a} = \frac{2}{\sigma} C_{Q_d} + \frac{2}{\sigma} C_{Q_j} \tag{6}$$

Equation (6) is now a quadratic equation in terms of $\theta$.

(5) Solve for $\theta$ from equation (6).

(6) With the known value of $\theta$, solve for $\lambda$ from equation (1).

(7) Solve for $\alpha$ from

$$\tan \alpha = \frac{\lambda}{\mu} + \frac{C_T}{2\mu(\lambda^2 + \mu^2)^{1/2}} \tag{7}$$

(8) Solve for $\left(\frac{D}{T}\right)_o$ from

$$\frac{\mu}{\cos \alpha} \frac{2C_T}{\sigma}\left(\frac{D}{T}\right)_o = 0.0087(t_{6,1}) - 0.0216\left[(t_{6,2})\lambda + (t_{6,3})\theta\right] + 0.400\left[(t_{6,5})\lambda^2 + (t_{6,6})\lambda\theta + (t_{6,8})\theta^2\right] \tag{8}$$

(9) Solve for $\left(\frac{D}{T}\right)_j$ from

$$\left(\frac{D}{T}\right)_j = \frac{A_j}{\pi R^2} \frac{\Delta c_{d_j}}{\mu C_T} \cos \alpha \left[\left(\frac{R_j}{R}\right)^3 + \frac{3}{2} \mu^2 \frac{R_j}{R}\right] \tag{9}$$

(10) Solve for $\left(\frac{D}{T}\right)_i$ from

$$\left(\frac{D}{T}\right)_i \doteq \frac{C_T \cos \alpha}{2\mu(\lambda^2 + \mu^2)^{1/2}} \tag{10}$$

(11) Solve for $\left(\frac{D}{T}\right)_p = \frac{f}{\pi R^2} \frac{\mu^2}{2C_T \cos^2 \alpha}$. (11)

(12) Solve for $\left(\frac{D}{T}\right)_g$ from the general performance equation as follows:

$$\left(\frac{D}{T}\right)_g = \left(\frac{D}{T}\right)_o + \left(\frac{D}{T}\right)_i + \left(\frac{D}{T}\right)_p + \left(\frac{D}{T}\right)_j \tag{12}$$

(13) An expression for the glide angle $\gamma$ in terms of the known values of $\left(\frac{D}{T}\right)_g$ and $\left(\frac{D}{T}\right)_p$ may be derived as follows:

With reference to figure 5, a summation of horizontal and vertical forces leads to the following relations:

$$T \sin i = D_p \cos \gamma \tag{13}$$

$$W = T \cos i + D_p \sin \gamma \tag{14}$$

If $D_g$ represents the drag force that absorbs the same amount of power at the gliding speed $V$ as supplied by the force of gravity during the glide, then

$$D_g V = WV \sin \gamma \tag{15}$$

or

$$\left(\frac{D}{T}\right)_g = \frac{W \sin \gamma}{T} \tag{16}$$

Substitution of equation (14) into equation (16) yields

$$\left(\frac{D}{T}\right)_g = \left[\cos i + \left(\frac{D}{T}\right)_p \sin \gamma\right] \sin \gamma \tag{17}$$

The expression for $\gamma$ is then obtained by solving equations (13) and (17) simultaneously

$$\sin \gamma = \frac{\left(\frac{D}{T}\right)_g}{\sqrt{1 - \left(\frac{D}{T}\right)_p^2 + 2\left(\frac{D}{T}\right)_p \left(\frac{D}{T}\right)_g}} \tag{18}$$

(14) Solve for $T$ from equation (16)

(15) Solve for $\Omega R$ from

$$\Omega R = \sqrt{\frac{T}{C_T \pi R^2 \rho}} \tag{19}$$

(16) Solve for $V$ from

$$V = \frac{\mu \Omega R}{\cos \alpha} \tag{20}$$

(17) Solve for $V_v$ and $V_h$ from

$$V_v = V \sin \gamma \tag{21}$$

$$V_h = V \cos \gamma \tag{22}$$

The contribution to the rate of descent brought about by any source of power loss may be found from equation (16). If, for example, it is desired to find that part of the total rate of descent contributed by the jet units at a given airspeed, then, replacing $\left(\frac{D}{T}\right)_g$ in equation (16) by $\left(\frac{D}{T}\right)_j$ yields

$$\left(\frac{D}{T}\right)_j = \frac{W}{T} \sin \gamma$$

$$= \frac{W}{T} \frac{V_v}{V} \tag{23}$$

and

$$V_v = \left(\frac{D}{T}\right)_j \frac{TV}{W} \tag{24}$$

## REFERENCES


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TABLE I

SUMMARY OF AUTOROTATIVE CHARACTERISTICS OF ASSUMED HELICOPTER

(a) $\frac{C_T}{\sigma} = 0.055$

| $\mu$ | $\lambda$ | $\theta$ (deg) | $\gamma$ (deg) | $\alpha$ (deg) | $V$ (mph) | $V_h$ (mph) | $V_v$ ft/min | $\Omega R$ (fps) | $T$ (lb) |
|---|---|---|---|---|---|---|---|---|---|
| $c_{d_j} = 0.20$ | | | | | | | | | |
| 0 | 0.09458 | -4.75 | 90.0 | 90.0 | 42.6 | 0 | 3,740 | 571 | 545 |
| .05 | .09497 | -4.78 | 68.1 | 65.1 | 45.9 | 17.1 | 3,750 | 567 | 537 |
| .10 | .09617 | -4.86 | 54.0 | 46.7 | 55.6 | 33.1 | 3,930 | 559 | 523 |
| .15 | .09857 | -5.03 | 47.4 | 35.3 | 68.5 | 46.4 | 4,435 | 546 | 499 |
| .20 | .10248 | -5.32 | 47.0 | 28.5 | 81.9 | 55.8 | 5,275 | 527 | 465 |
| .25 | .10918 | -5.82 | 50.0 | 24.6 | 94.2 | 60.5 | 6,350 | 503 | 422 |
| .30 | .12170 | -6.77 | 54.6 | 22.8 | 104.7 | 60.6 | 7,510 | 472 | 372 |
| $c_{d_j} = 0.10$ | | | | | | | | | |
| 0 | 0.05449 | -1.20 | 90.0 | 90.0 | 31.6 | 0 | 2,780 | 582 | 566 |
| .05 | .05446 | -1.20 | 58.2 | 55.6 | 35.2 | 18.5 | 2,630 | 582 | 566 |
| .10 | .05434 | -1.19 | 40.2 | 33.6 | 47.2 | 36.1 | 2,680 | 577 | 556 |
| .15 | .05423 | -1.19 | 35.3 | 23.7 | 63.2 | 51.6 | 3,200 | 566 | 536 |
| .20 | .05409 | -1.19 | 37.3 | 16.9 | 78.4 | 62.1 | 4,180 | 550 | 505 |
| .25 | .05417 | -1.22 | 41.7 | 13.4 | 92.3 | 68.9 | 5,400 | 527 | 464 |
| .30 | .05435 | -1.26 | 47.8 | 11.1 | 103.8 | 69.7 | 6,765 | 498 | 414 |
| $c_{d_j} = 0$ | | | | | | | | | |
| 0 | 0.02004 | 1.86 | 90.0 | 90.0 | 25.8 | 0 | 2,270 | 588 | 577 |
| .05 | .01974 | 1.87 | 44.5 | 42.3 | 27.2 | 19.4 | 1,680 | 591 | 583 |
| .10 | .01882 | 1.92 | 24.3 | 17.9 | 42.2 | 38.5 | 1,525 | 589 | 579 |
| .15 | .01704 | 2.03 | 22.8 | 9.9 | 60.4 | 55.7 | 2,055 | 582 | 565 |
| .20 | .01492 | 2.15 | 27.9 | 6.2 | 77.8 | 68.8 | 3,200 | 567 | 537 |
| .25 | .01185 | 2.32 | 34.7 | 4.0 | 93.2 | 76.7 | 4,665 | 545 | 497 |
| .30 | .00784 | 2.55 | 42.7 | 2.4 | 102.9 | 75.6 | 6,140 | 503 | 422 |

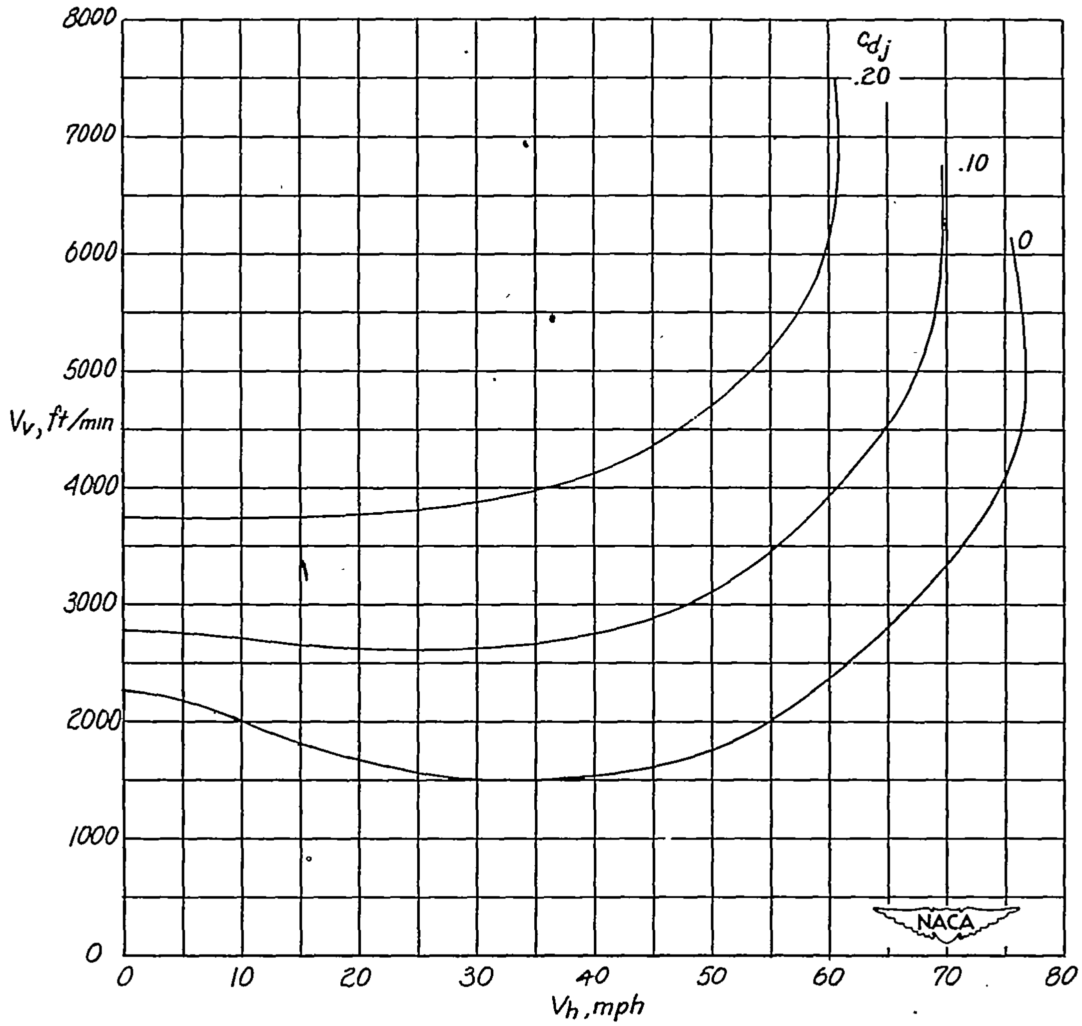
TABLE I

SUMMARY OF AUTOROTATIVE CHARACTERISTICS OF ASSUMED HELICOPTER - Concluded

(b) $\frac{C_T}{\sigma} = 0.124$

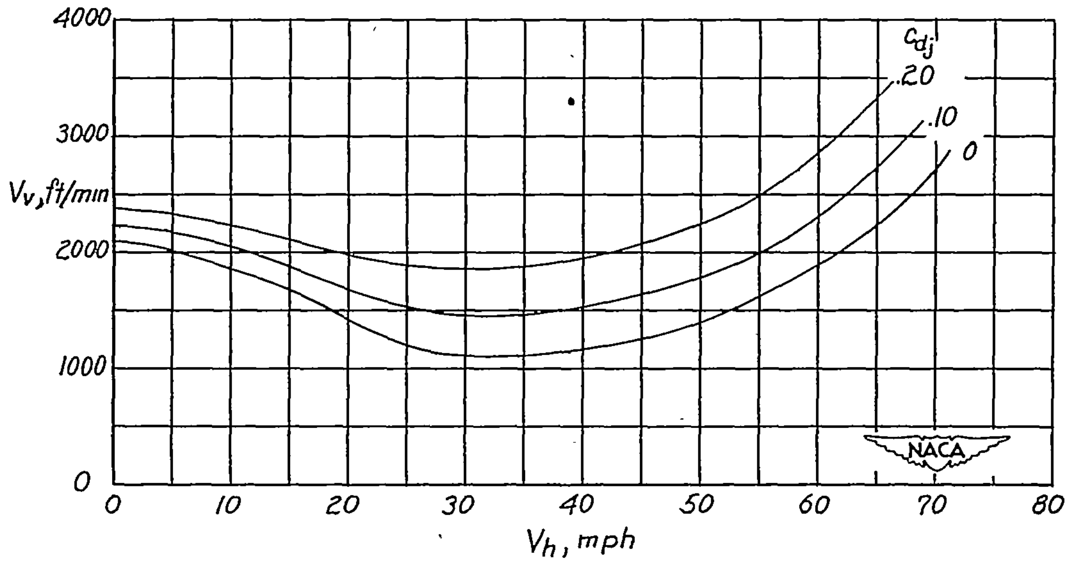
| $\mu$ | $\lambda$ | $\theta$ (deg) | $\gamma$ (deg) | $\alpha$ (deg) | V (mph) | $V_h$ (mph) | $V_v$ ft/min | $\Omega R$ (fps) | T (lb) |
|---|---|---|---|---|---|---|---|---|---|
| $c_{d_j} = 0.20$ | | | | | | | | | |
| 0 | 0.04602 | 4.07 | 90.0 | 90.0 | 27.0 | 0 | 2,380 | 391 | 575 |
| .05 | .04517 | 4.13 | 63.5 | 61.3 | 27.8 | 12.4 | 2,190 | 392 | 578 |
| .10 | .04303 | 4.25 | 40.4 | 35.6 | 32.9 | 25.1 | 1,875 | 392 | 578 |
| .15 | .03912 | 4.49 | 30.1 | 21.5 | 42.9 | 37.1 | 1,895 | 391 | 572 |
| .20 | .03370 | 4.80 | 27.1 | 13.8 | 54.4 | 48.4 | 2,180 | 387 | 563 |
| .25 | .02635 | 5.24 | 27.8 | 8.8 | 65.9 | 58.3 | 2,705 | 382 | 547 |
| .30 | .01682 | 5.79 | 30.7 | 5.2 | 77.1 | 66.3 | 3,460 | 375 | 528 |
| $c_{d_j} = 0.10$ | | | | | | | | | |
| 0 | 0.02867 | 5.61 | 90.0 | 90.0 | 25.6 | 0 | 2,245 | 393 | 578 |
| .05 | .02775 | 5.67 | 60.9 | 58.6 | 25.8 | 12.6 | 1,980 | 394 | 581 |
| .10 | .02537 | 5.80 | 33.8 | 29.0 | 30.7 | 25.6 | 1,505 | 394 | 583 |
| .15 | .02089 | 6.07 | 24.0 | 15.4 | 41.7 | 28.1 | 1,495 | 393 | 579 |
| .20 | .01468 | 6.43 | 22.1 | 8.6 | 53.8 | 49.9 | 1,780 | 390 | 571 |
| .25 | .00621 | 6.92 | 23.7 | 4.3 | 65.9 | 60.4 | 2,335 | 386 | 558 |
| .30 | -.00443 | 7.53 | 27.4 | 1.1 | 77.5 | 68.9 | 3,135 | 379 | 539 |
| $c_{d_j} = 0$ | | | | | | | | | |
| 0 | 0.01363 | 6.94 | 90.0 | 90.0 | 23.8 | 0 | 2,100 | 394 | 581 |
| .05 | .01261 | 7.01 | 57.7 | 55.5 | 23.8 | 12.7 | 1,770 | 395 | 585 |
| .10 | .00997 | 7.15 | 26.9 | 22.0 | 29.1 | 26.0 | 1,160 | 396 | 588 |
| .15 | .00508 | 7.44 | 18.4 | 9.7 | 41.0 | 38.9 | 1,140 | 395 | 585 |
| .20 | -.00170 | 7.82 | 17.7 | 4.0 | 53.7 | 51.1 | 1,435 | 393 | 578 |
| .25 | -.01091 | 8.35 | 20.2 | .3 | 22.9 | 62.2 | 2,015 | 389 | 567 |
| .30 | -.02252 | 9.01 | 24.6 | -2.3 | 32.5 | 71.2 | 2,860 | 382 | 548 |

NACA




(a) Approximate $\overline{C}_L = 0.33$; $\frac{C_T}{\sigma} = 0.055$.

Figure 1.- Effect of tip-jet drag on autorotative performance of assumed helicopter.




(b) Approximate $\overline{C}_L = 0.74$; $\frac{C_T}{\sigma} = 0.124$.

Figure 1.- Concluded.

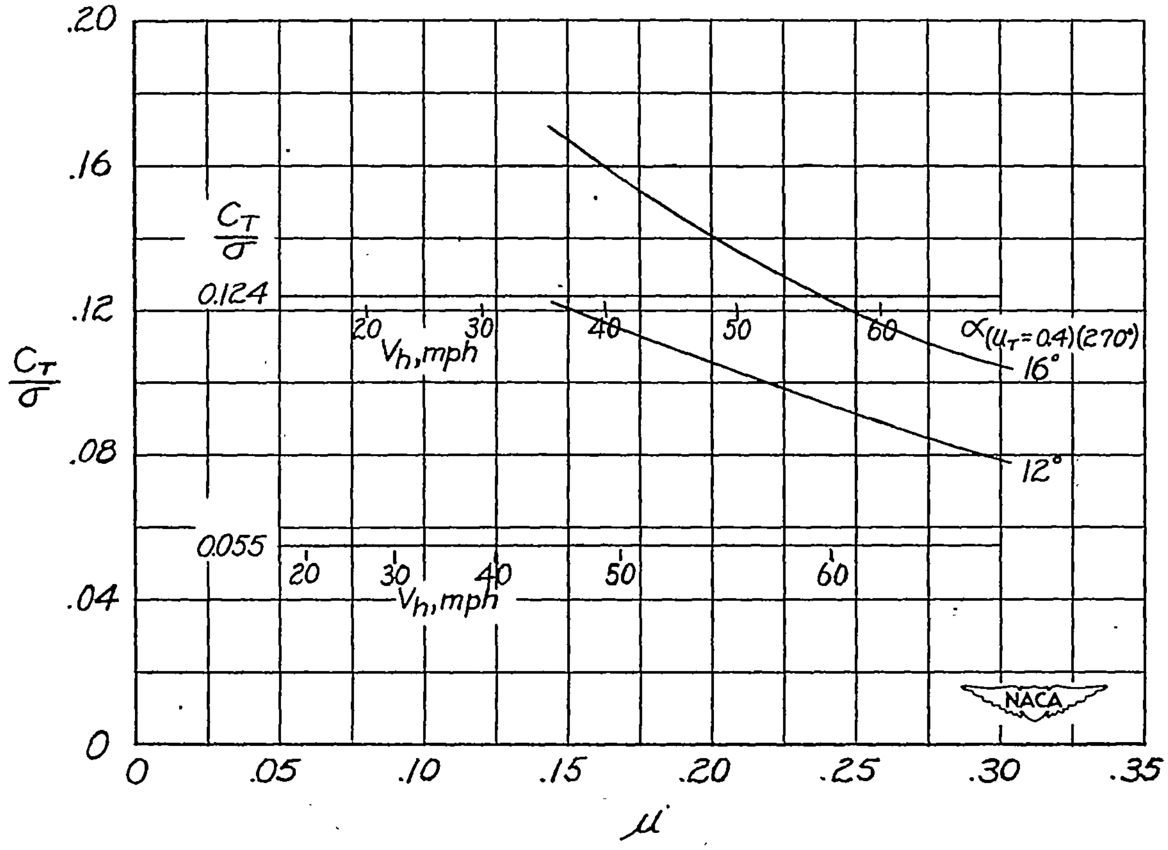



Figure 2.- Variation of limiting $C_T/\sigma$ ratios, as limited by inboard stall, with $\mu$.

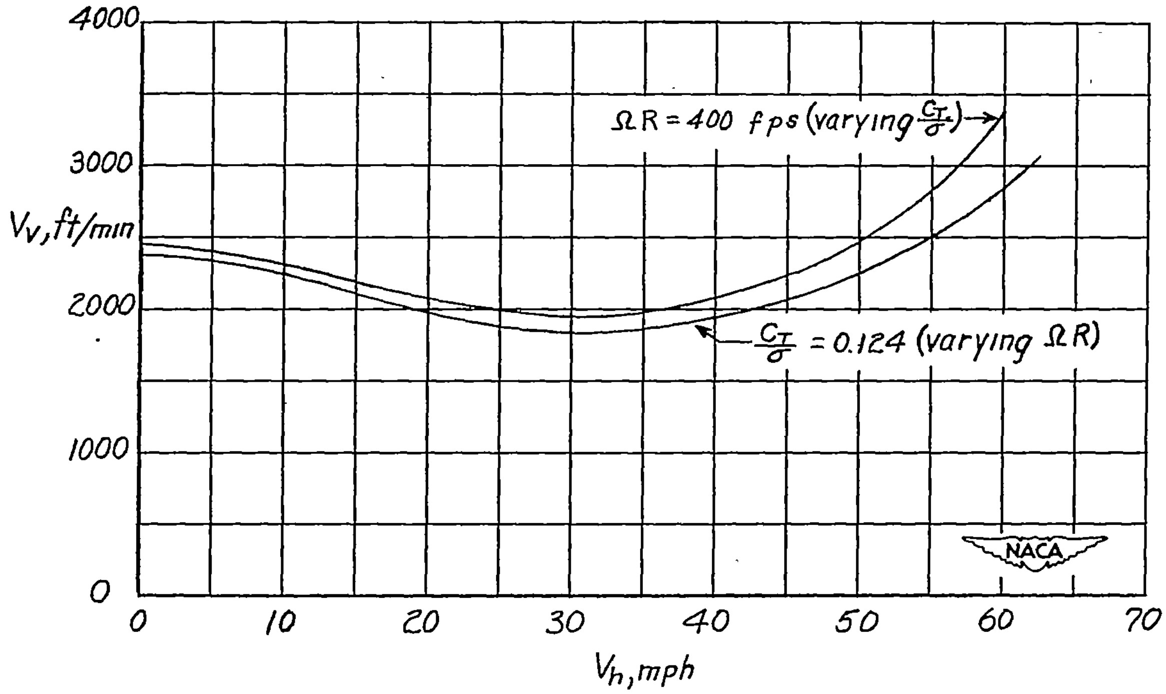
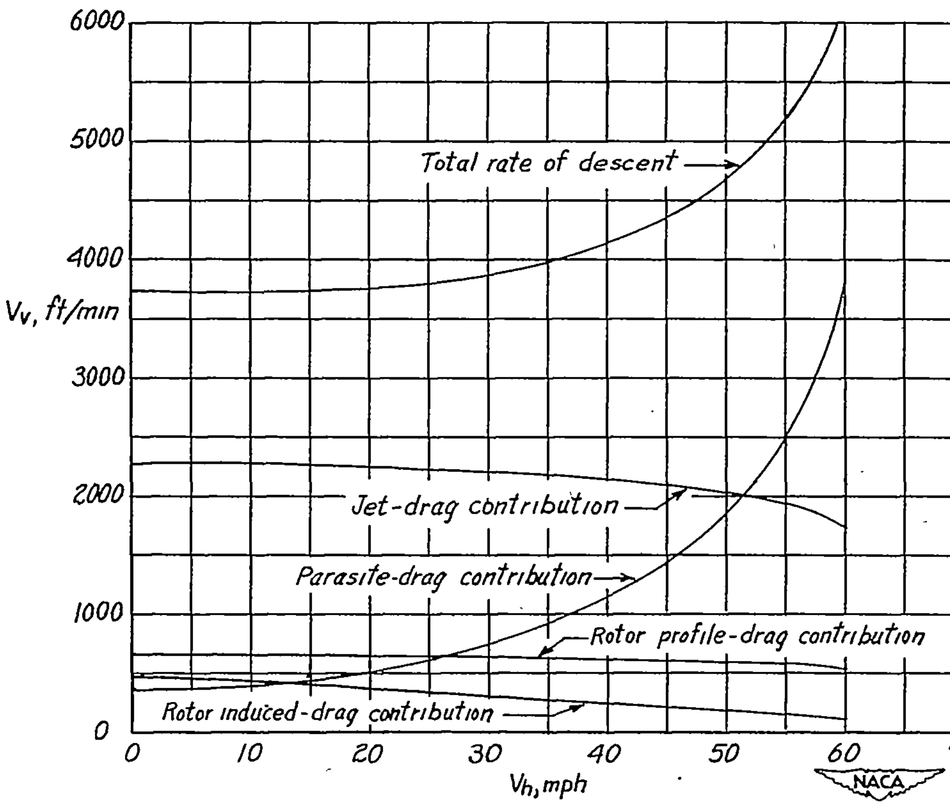
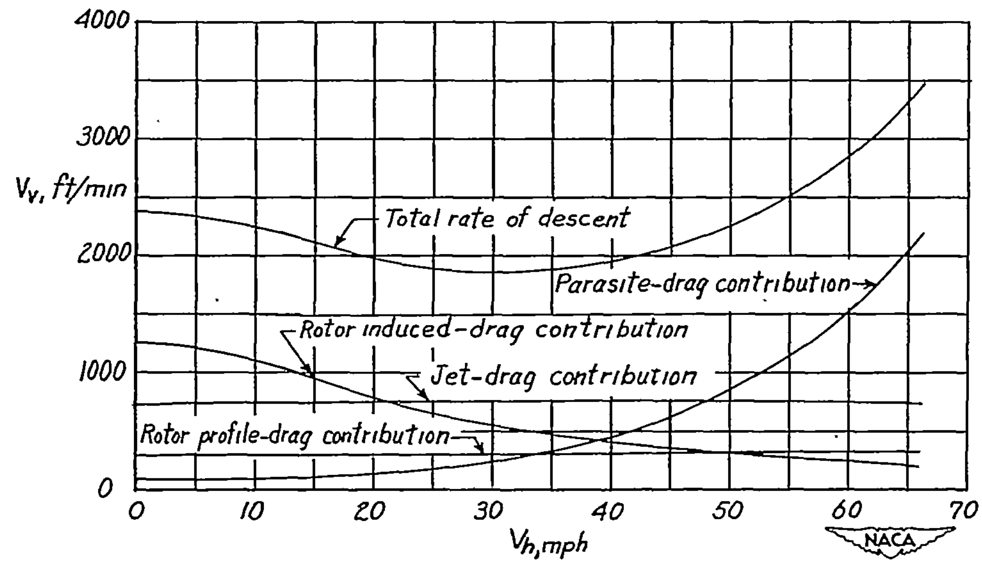



Figure 3.- Effect on autorotative performance of assumed helicopter of operating at constant tip speed as compared with constant mean lift coefficient; $c_{d_j} = 0.20$.




(a) Approximate $\bar{C}_L = 0.33$; $\frac{C_T}{\sigma} = 0.055$.

Figure 4.- Contributions of individual rotor and fuselage losses to total autorotative rate of descent of assumed helicopter; $c_{d_j} = 0.20$.




(b) Approximate $\overline{C_L} = 0.74$; $\frac{C_T}{\sigma} = 0.124$.

Figure 4.- Concluded.

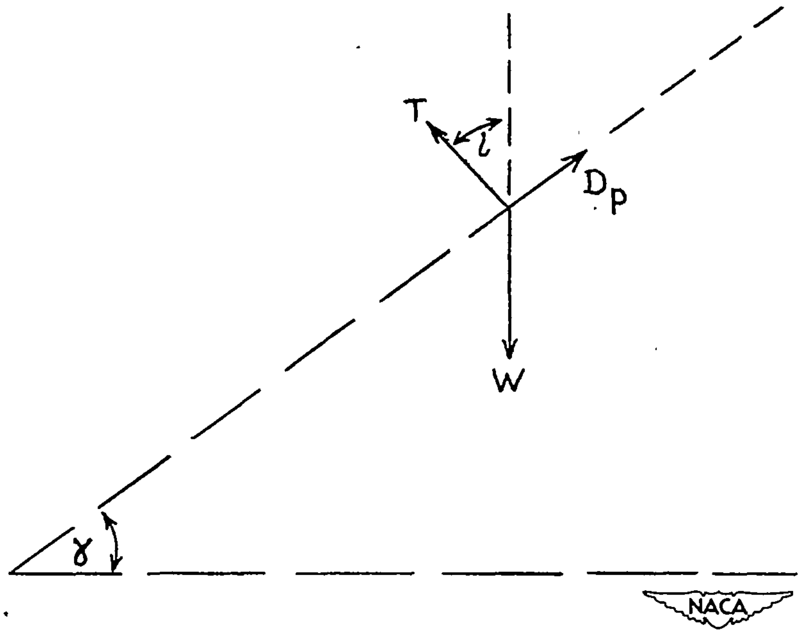



Figure 5.- Diagram of forces acting on a helicopter in a glide.